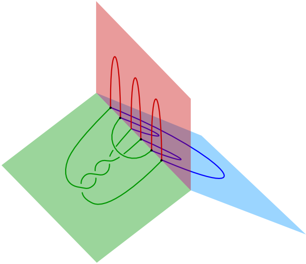
Trisections and Surgery



Jeffrey Meier

Department of Mathematics, Indiana University

May 26, 2016

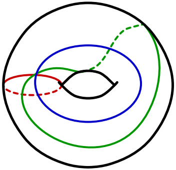
Joint with David Gay (University of Georgia)
and Alexander Zupan (University of Nebraska, Lincoln)

This research was supported in part by NSF grants DMS-1400543.

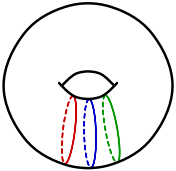
Trisection diagrams

Theorem (Gay–Kirby, 2012)

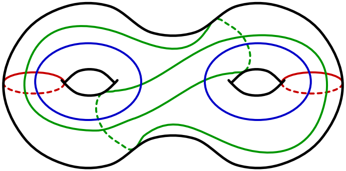
Every closed, orientable, smooth four-manifold can be described by a trisection diagram, which is unique up to a calculus of moves.



$\mathbb{CP}^2$



$S^1 \times S^3$



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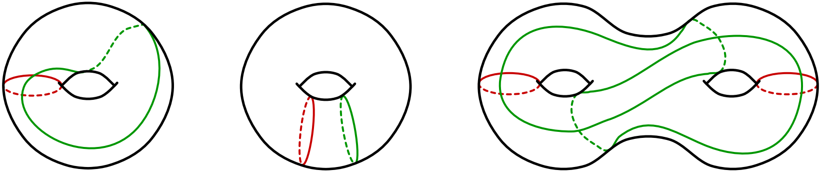
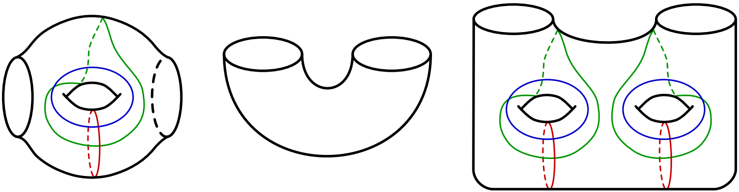


Relative trisections

Theorem (Castro–Gay–Pinzón-Caicedo, 2016)

Every orientable, smooth four-manifold with boundary can be described by a relative trisection diagram, which is unique up to a calculus of moves.

Moreover, the diagram determines an open book decomposition of the boundary three-manifold.



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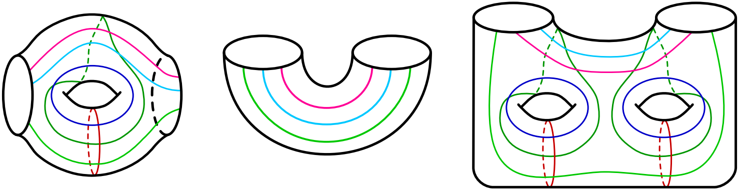
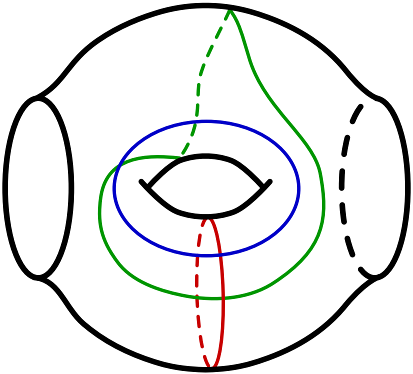


Example with the four-ball



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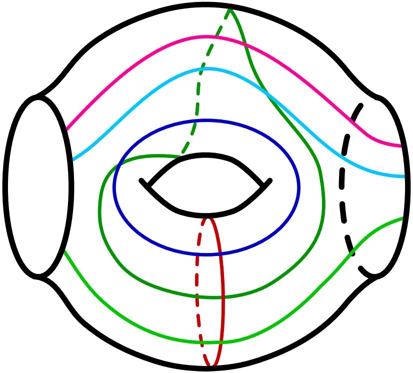

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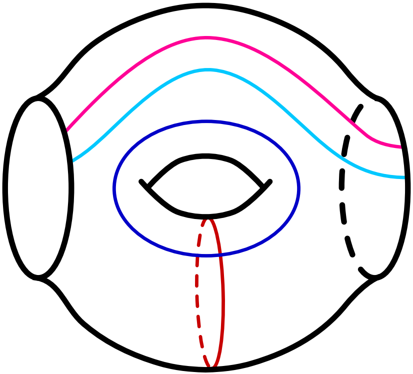

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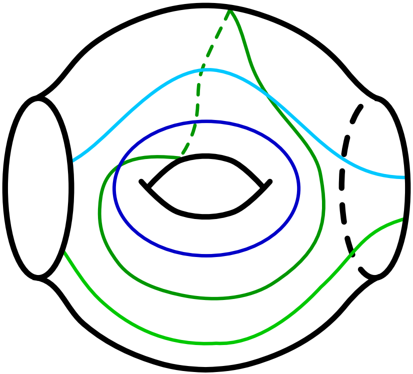

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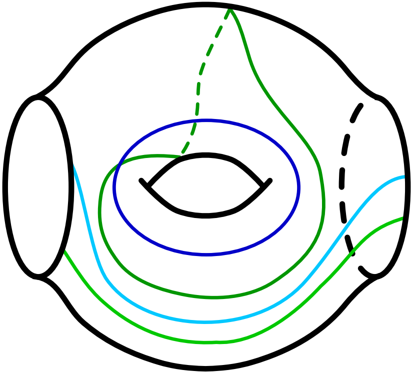

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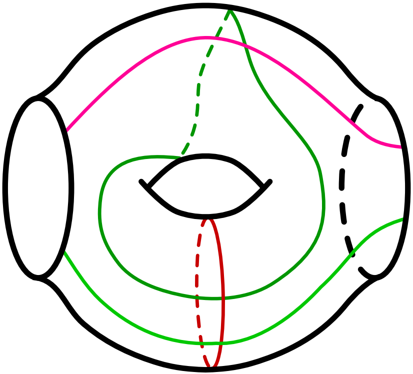

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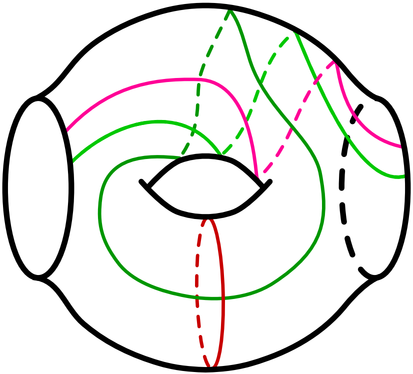
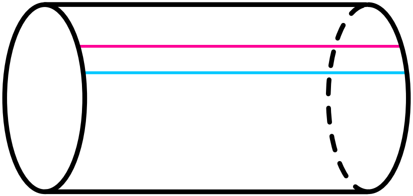


The boundary open book



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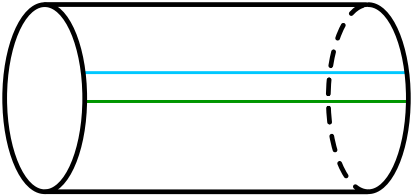

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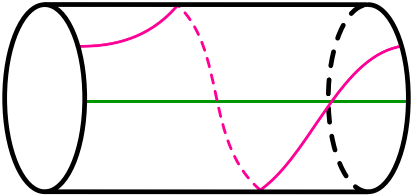
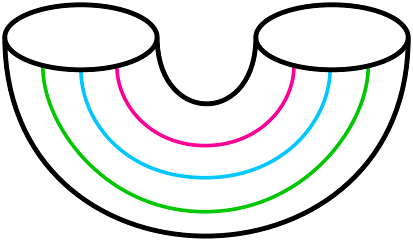


GLUING $S^1 \times B^3$ TO GET $S^1 \times S^3$



$$S^1 \times B^3$$

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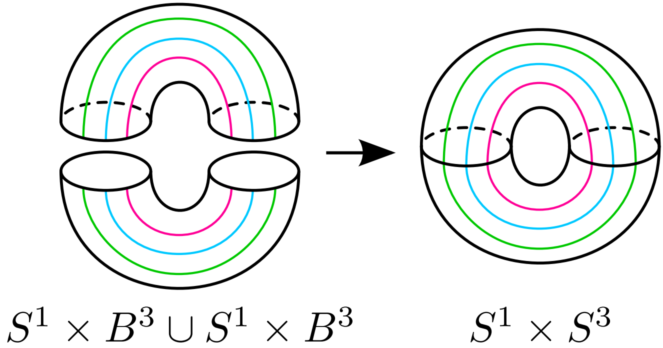
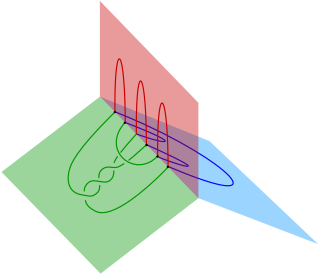
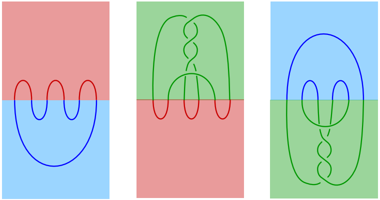


Bridge trisections

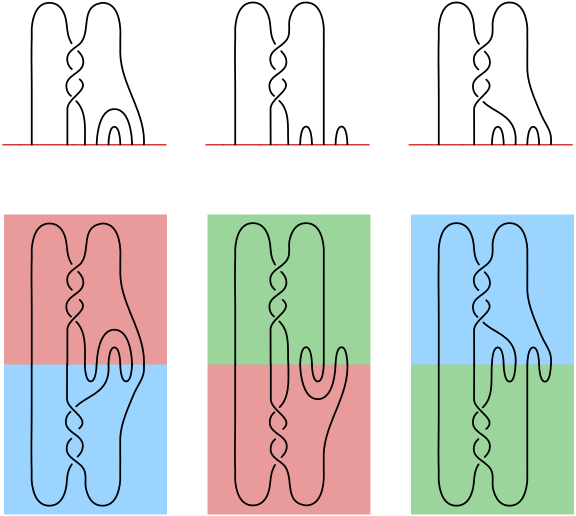
Theorem (Meier–Zupan, 2015)

Every knotted surface in the four-sphere can be described by a tri-plane diagram, which is unique up to a calculus of moves.





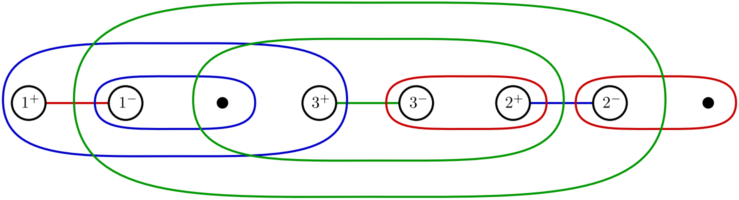
The spun trefoil



Doubly-pointed trisection diagrams

Theorem

Let $\mathcal{K}$ be a 2–knot in a four-manifold X. Then, the pair $(X, \mathcal{K})$ can be described by a doubly-pointed trisection diagram.



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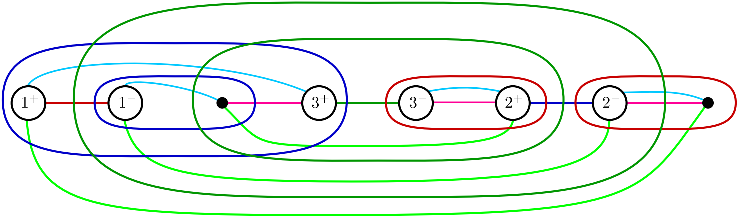
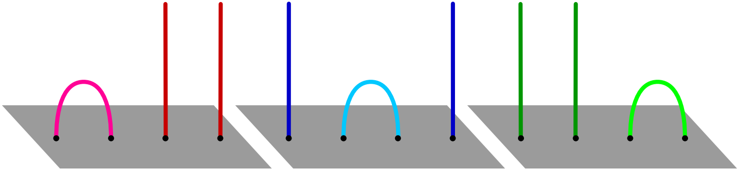


Meridional stabilization



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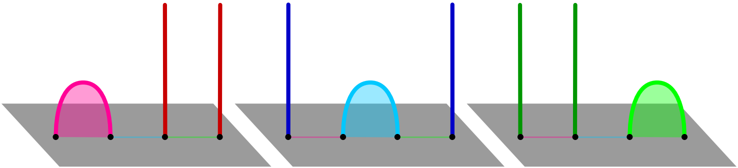

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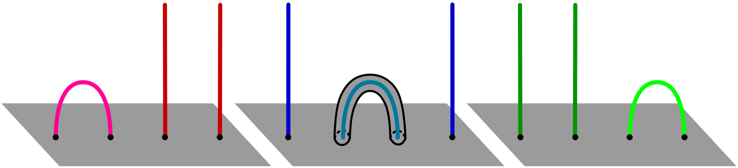

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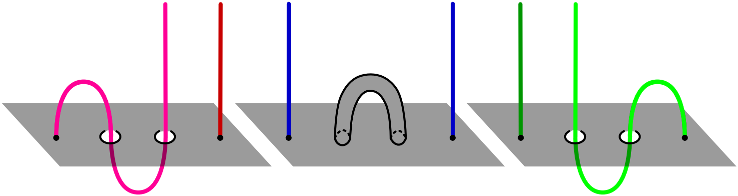

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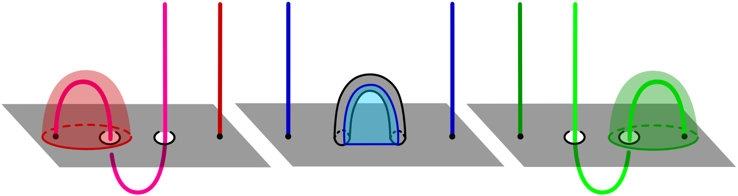
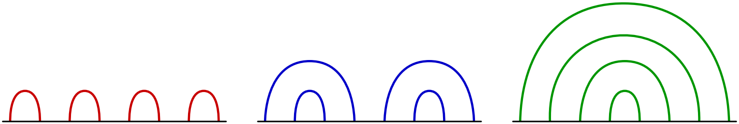


Example with the unknot



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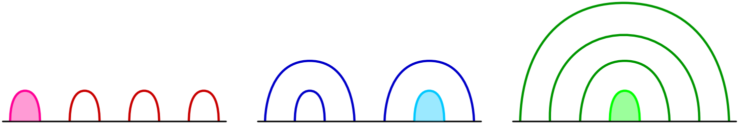

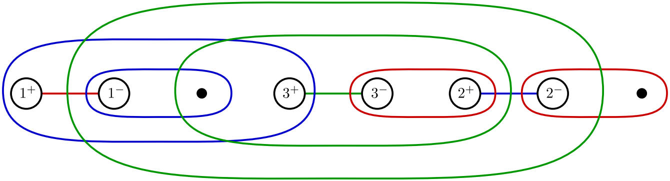

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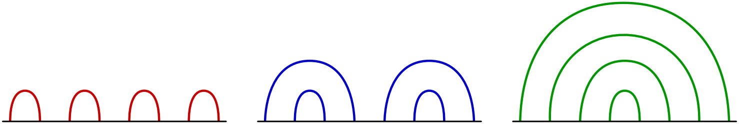
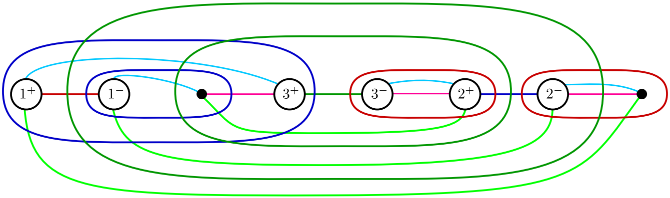
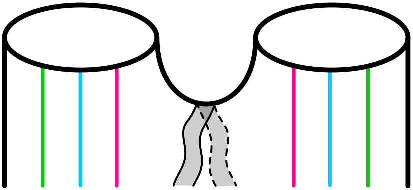




2–knot complements

Theorem

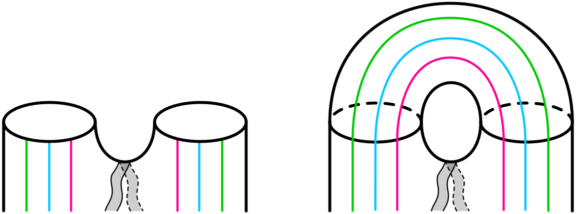
Let $\mathcal{K}$ be a 2–knot in a four-manifold X. Then the complement $X \setminus \nu(\mathcal{K})$ can be described by a relative trisection diagram with two boundary components, which has the property that the three arcs are pairwise equivalent.



Surgery

Theorem

Let $\mathcal{K}$ be a 2–knot in a four-manifold X, and let $X_0(\mathcal{K})$ denote the result of performing surgery on $\mathcal{K}$. If $\mathcal{T}$ is a relative trisection diagram for $X \setminus \nu(\mathcal{K})$, then $\overline{\mathcal{T}}$ is a diagram for $X_0(\mathcal{K})$:

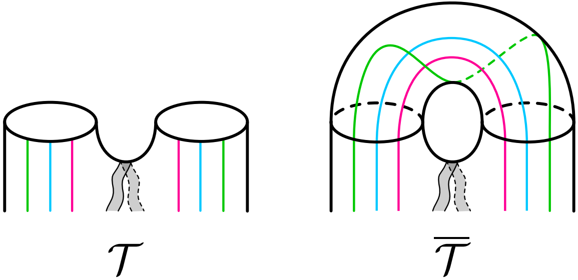


$\mathcal{T}$ $\overline{\mathcal{T}}$

Gluck twist

Theorem

Let $\mathcal{K}$ be a 2–knot in a four-manifold X, and let $\mathcal{G}(X, \mathcal{K})$ denote the result of performing a Gluck twist on $\mathcal{K}$. If $\mathcal{T}$ is a relative trisection diagram for $X \setminus \nu(\mathcal{K})$, then $\overline{\mathcal{T}}$ is a diagram for $\mathcal{G}(X, \mathcal{K})$:



Thank You!